Electron acceleration and high-order harmonic generation by an intense short pulse laser in a magnetic field

Wei Yu,[1,2] Z. Y. Chen,[2] M. Y. Yu,[2] L. J. Qian,[1] P. X. Lu,[1] R. X. Li,[1] K. Koyama[3]
[1]*Shanghai Institute of Optics and Fine Mechanics, Shanghai 201800, People's Republic of China*
[2]*Institut für Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[3]*National Institute of Advanced Industrial Science and Technology, Tsukuba, Ibaraki 305-8568, Japan*
(Received 30 April 2002; published 24 September 2002)

Electron acceleration and short-wavelength radiation generation by an intense, short, linearly polarized laser pulse in an external magnetic field parallel to that of the light wave is investigated classically. It is found that enhanced electron acceleration by the laser can occur. After the pulse has passed, much of the energy gained by the electron is retained in its relativistic cyclotron motion. The electron emits radiation at high harmonics of the cyclotron frequency until the static magnetic field decays.

DOI: 10.1103/PhysRevE.66.036406 PACS number(s): 52.38.Kd, 41.75.Jv, 42.65.Ky, 71.15.Rf

I. INTRODUCTION

The appearance of ultraintense ($a=eA/mc^2 \geqslant 1$, where A is the vector potential, e and m are the electron charge and mass, and c is the speed of light) lasers has led to much interest in the possibility of electron acceleration and harmonic generation in laser-plasma interactions [1–19]. Schemes based on laser-induced relativistic effects have been proposed for accelerating electrons to very high energies and for generating very short-wavelength radiation. An electron in an intense laser pulse can not only reach relativistic quiver speeds, but also be accelerated to such speeds in the direction of laser propagation by the ponderomotive force at the pulse front. An electron executing relativistic quiver motion can emit radiation at high harmonics of the laser frequency. The total energy an electron can gain or radiate depends strongly on the interaction time of the electron in the laser pulse.

Wagner, Su, and Grobe (WSG) [17] proposed a scheme for accelerating electrons to the relativistic regime with a laser orders of magnitude weaker than that of most other schemes. Using quantum theory, they showed that with a laser pulse of $a<0.1$ and a static magnetic field of suitable magnitude and alignment, atomic electrons can be ionized and accelerated to relativistic speeds. The use of external magnetic fields to modify the efficiency of accelerating an electron in vacuum by a focused laser has been considered earlier by Apollonov and coworkers [20–22] for less intense lasers. They found that under suitable conditions, the oscillating wave field can be partially masked by the external field, so that acceleration takes place in a quasistatic manner [23] if the focal region is sufficiently long. In the present paper, we reexamine the WSG mechanism using a classical mechanics approach and determine the relativistic electron dynamics in the laser and a static magnetic field parallel to the wave magnetic field, so that both the wave electric field and the ponderomotive field will affect the cyclotron motion of the electron. It is shown that besides energization by the ponderomotive force, an ionized electron can also gain much energy if the phase in its cyclotron motion with respect to the laser field oscillation at the instant of ionization is appropriate. The maximum velocity such an electron can achieve as well as its dependence on the magnetic field strength for $a<0.1$ are investigated. The resulting single-sawtooth profile in the maximum speed *versus* magnetic field curve agrees well with that of WSG [17], indicating that the physics of the WSG mechanism is mainly classical. That is, the effect of the atomic potential plays a minor role in the evolution of the electron motion.

Extending our calculation to higher laser intensities ($a \geqslant 0.1$), we find that the sawtooth is replaced by a jagged flat-top profile, and acceleration of an electron to relativistic speeds can now also occur for weaker magnetic fields. More interestingly, an intense external magnetic field parallel to the laser magnetic field can not only enhance the electron acceleration, but also lead to retention of much of the energy gained by the electron from the combined actions of the laser and the magnetic field after the passing of the laser pulse. Furthermore, since a considerable part of the electron acceleration occurs in its circular (cyclotron) motion, the linear acceleration distance remains very small. Present-day high-intensity magnetic fields are of short duration, but still several orders of magnitude longer than that of the laser pulse. Thus, the energized electron left behind by the pulse will still be inside the magnetic field and continues its relativistic cyclotron motion. High harmonics of the electron cyclotron frequency are thus emitted until the magnetic field decays. The radiation time is then given by the lifetime of the quasistatic magnetic field instead of the very short duration of a high-intensity laser pulse.

II. FORMULATION

The equations for the relativistic electron motion in an electromagnetic field can be written as

$$d_t(\mathbf{p}-\mathbf{a}) = -\tilde{\boldsymbol{\nabla}}(\mathbf{a}\cdot\mathbf{u}), \tag{1}$$

$$d_t\gamma = \mathbf{u}\cdot\partial_t\mathbf{a}, \tag{2}$$

where $d_t=\partial_t+\mathbf{u}\cdot\boldsymbol{\nabla}$, $\mathbf{a}$ is the vector potential normalized by mc^2/e, $\mathbf{u}$ is the electron velocity normalized by $c, \mathbf{p}=\gamma\mathbf{u}$ is the normalized electron momentum, $\gamma=(1-u^2)^{-1/2}$ is the relativistic factor or normalized energy, the time t is normalized by ω^{-1}, the space coordinates are normalized by k^{-1}, and ω and k are the laser frequency and wave number, respectively. The operator $\tilde{\nabla}$ in Eq. (2) acts only on $\mathbf{a}$.

We shall assume that the pulse is linearly polarized and the static magnetic field is parallel to the wave magnetic field. Accordingly, the vector potential can be written as $\mathbf{a} = \mathbf{a}_L + \mathbf{a}_0$, where $\mathbf{a}_L = \hat{x} a_L(\eta)$ is the envelope of the laser vector potential and $\mathbf{a}_0 = \hat{x} B_0 z$ is the vector potential of the static magnetic field. Here, $\eta = z - t$ is the frame moving with the laser pulse at nearly the speed of light, and $\mathbf{B}_0 = \hat{y} B_0$ is the external magnetic field normalized by $mc\omega/e$. In the one-dimensional case, Eqs. (1) and (2) yield

$$\gamma u_x = a - a_i, \tag{3}$$

$$d_t(\gamma u_z) = -(B_0 + d_\eta a_L) u_x, \tag{4}$$

$$d_t \gamma = -u_x d_\eta a_L, \tag{5}$$

where a_i is the vector potential at which the electron is ionized (at zero velocity), and we have noted that $a_L = a_L(\eta)$. Furthermore, we have

$$\gamma^{-2} = 1 - u_x^2 - u_z^2 = (1 - u_z^2)/[1 + (a - a_i)^2], \tag{6}$$

and get from Eqs. (3)–(6) the governing equation

$$d_t u_z = -(1 - u_z^2)(a - a_i)\frac{B_0 + (1 - u_z) d_\eta a_L}{1 + (a - a_i)^2}, \tag{7}$$

for the longitudinal motion of the electron.

We consider a Gaussian laser pulse propagating in an atomic gas with $a_L = a_0 \exp[-(\eta - z_0)^2/L^2]\cos(\eta)$, where a_0 is the normalized peak amplitude and L is the normalized pulse width of the laser. An atom is assumed to be ionized when the local laser strength a_L exceeds the (normalized) ionization potential I_{Coul} according to the Coulomb-barrier model. For definitiveness we assume that the trajectory of an ionized electron starts at $t = 0$ and $z = 0$, when the peak of the laser pulse is at $z = z_0 < 0$, which is determined by the laser strength at ionization, or when $a_i = I_{\mathrm{Coul}}$. The classical trajectory of the ionized electron is then fully determined by Eq. (7), which is highly inhomogeneous and will be solved numerically.

III. NUMERICAL RESULTS

Figure 1 shows the time evolution of the normalized energy γ of an ionized electron for $L = 100$ and different laser and magnetic field strengths, as well as that of the laser at ionization: (a) $a_0 = 0.08$, $B_0 = 1.2$, $a_i = 0.047$; (b) $a_0 = 0.5$, $B_0 = 0.5$, $a_i = 0.05$; and (c) $a_0 = 5$, $B_0 = 0.3$, $a_i = 0.3$. One sees that the electron energy increases under the combined action of the laser and magnetic fields in a complex oscillatory manner. Besides that by the ponderomotive force, the acceleration also depends sensitively on the phase between the laser electric field and the state of the electron in its cyclotron orbit. When the laser pulse eventually overtakes the electron, the latter retains a considerable portion of the gained energy. The energy remains constant as long as the magnetic field is unchanged, as shown in the figures at large t. That is, the combined effect of the laser and static magnetic field leads not only to electron acceleration but also to retention of most of the gained energy after the passing of the pulse. As a result, significant net energy gain by the ionized electron is achieved.

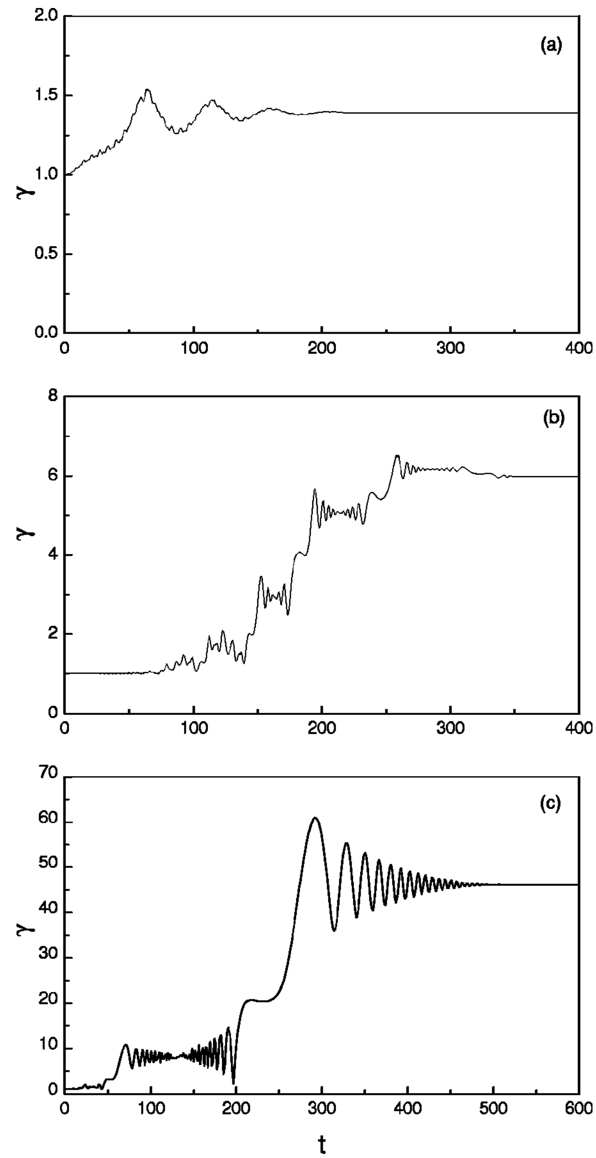

FIG. 1. The time evolution of the electron energy γ for $L = 100$, (a) $a_0 = 0.08$, $B_0 = 1.2$, and $a_i = 0.047$; (b) $a_0 = 0.5$, $B_0 = 0.5$, and $a_i = 0.05$; and (c) $a_0 = 5$, $B_0 = 0.3$, and $a_i = 0.3$.

Without the static magnetic field, the energy of an ionized electron in a planar laser pulse would be

$$\gamma = 1 + (a - a_i)^2/2, \tag{8}$$

which yields the maximum energy $\gamma_m = 1 + (a_0 - a_i)^2/2$ near the center of the pulse and the escape energy $\gamma_f = 1 + a_i^2/2$. Comparing with Eq. (6), we see that both of these energies are far less than that in the presence of the static magnetic

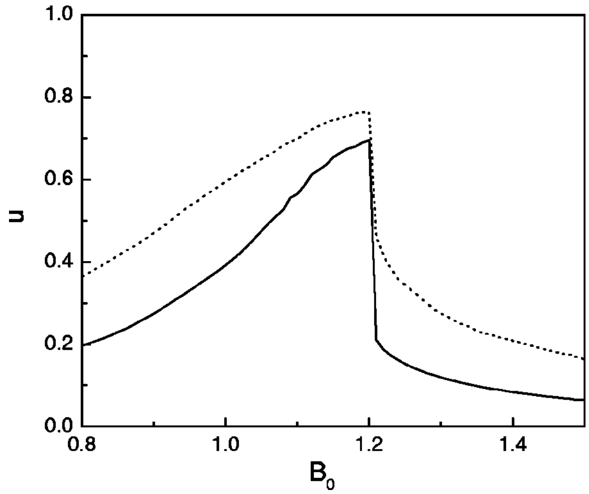

FIG. 2. The maximum electron velocity (dotted curve) and maximum escape velocity (solid curve) as functions of B_0, where $L=100$ and $a_0=0.08$.

field. This is because in the absence of the latter, much of the energy gained in the rising front of the laser pulse is lost in the trailing part.

The maximum speed $u_{\max}=(1-\gamma_{\max}^{-2})^{1/2}$ which an electron can ever achieve from the combined action of the laser and magnetic fields as a function of B_0 is given by the dotted curve in Fig. 2, for $L=100$ and $a_0=0.08$. One sees that this curve is nearly identical to Fig. 1 of WSG [17], particularly with respect to the unique sawtoothlike peak at $B_0\sim 1.2$ (with a maximum speed of up to 0.8). This agreement is expected since for most of its trajectory the electron is far from the parent atom so that the effect of the atomic potential [17] should be weak. The maximum escape velocity, obtained by examining all possible a_i's, is given in Fig. 2 by the solid curve. We see that the B_0 dependence has again a sawtoothlike peak.

For higher laser intensities the scenario becomes quite different. Figure 3 is for $L=100$, and (a) $a_0=0.5$ and (b) $a_0=5$. We see that at higher intensities ($a_0\gg 0.1$) the sawtooth profile at $B_0\sim 1$ is replaced by a wide jagged flat-top profile with its width extending from $B_0\ll 1$ to $B_0>1$. That is, in the present scheme, effective acceleration can occur for weaker magnetic fields than that of WSG [17], as can also been seen in Figs. 1(b) and 1(c) for $B_0=0.5$ and 0.3, respectively. We note that Figs. 2 and 3 are for optimized ionization phases. For the average electron the maximum and escape velocities would be somewhat smaller but of the same order. On the other hand, the trajectories shown in Fig. 1 do not correspond to optimized ionization phases.

The duration of present-day intense magnetic fields is very short but still several orders of magnitude longer than that of a short laser pulse. Energized electrons left behind by the laser pulse will continue to execute cyclotron motion in the static magnetic field. Their trajectories are given by Eqs. (3) – (7) with $a_L=0$. One can show that the electron orbit is centered at $x=0$ and $z=-a_i/B_0$, with a gyration radius $\rho=(\gamma_f/B_0)k^{-1}$. Although the electron has experienced a huge energy increase since its ionization, its actual displacement is confined to a very small region, ensuring that it remains inside the magnetic field. In the present scheme the electron is energized mainly in its cyclotron motion, so that the acceleration distance, of the order of the laser wavelength, is much smaller than that in other laser acceleration schemes.

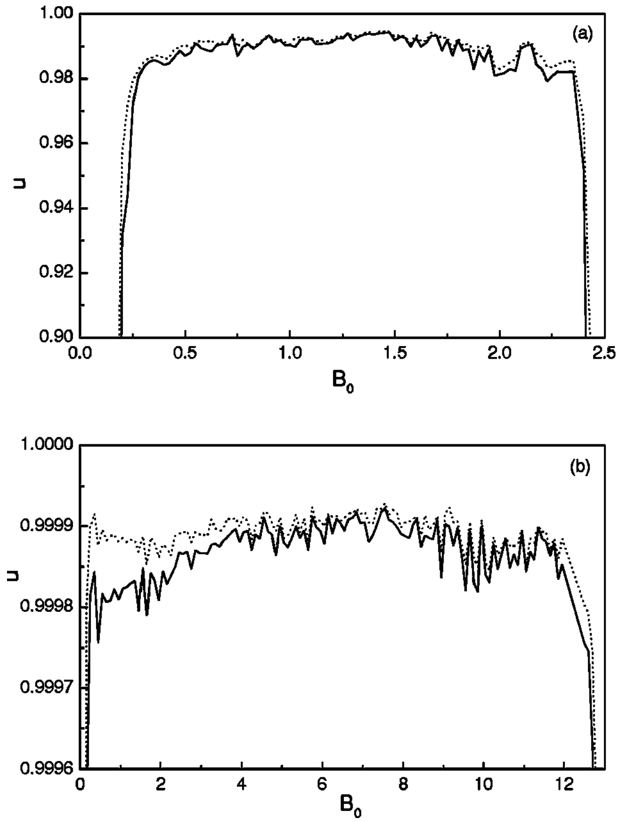

FIG. 3. The maximum electron velocity (dotted curve) and maximum escape velocity (solid curve) as functions of B_0 for $L=100$, $a_0=0.5$ (a) and 5.0 (b).

IV. HIGH-HARMONIC GENERATION

WSG [17] also investigated the emission of high harmonics by the laser-driven relativistic electrons. In general, a relativistic electron gyrating with a frequency $\Omega=c/\rho$ emits radiation at the harmonics of the latter. The radiated power per solid angle for the $n\Omega$th harmonic can be written as [10,24]

$$d_\theta P_n=\frac{\hbar n^2\Omega^2}{2\pi\alpha}[\cot^2\theta J_n(n\sin\theta)+u_f^2\dot{J}_n^2(n\sin\theta)], \quad (9)$$

where $\alpha=1/137$, θ is the angle of observation measured from the normal of the circular orbit, and J_n and $\dot{J}_n$ are the nth-order Bessel function and its derivative, respectively. In this process, an appreciable amount of the emitted radiation is in the high-harmonic components. The latter can be, up to the critical frequency, given by

$$\Omega_c=1.5\gamma_f^3\Omega, \quad (10)$$

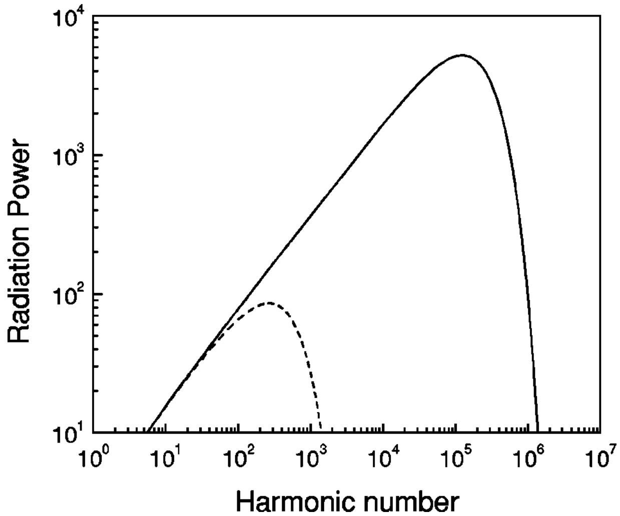

FIG. 4. The n dependence of radiated power $d_\theta P_n/P$ observed in the plane of the electron cyclotron orbit ($\theta=\pi/2$). The other parameters are the same as in Figs. 1(b) (dashed curve) and 1(c) (solid curve), respectively.

and the total radiated power can be expressed as

$$P=(2\gamma_f^4/3\alpha)(\hbar\Omega^2), \quad (11)$$

so that for obtaining radiation at short wavelength and high power, it is desirable to have a large γ_f and small ρ. It is of interest to point out that Eqs. (9) – (11) are similar to the formulas describing synchrotron radiation. This is expected since electrons driven by a circularly polarized, ultraintense laser radiate in a similar manner as that in a synchrotron [1,10,24].

In the combined laser–magnetic field scheme discussed here, the radiation will continue for a much longer time after the passing of the laser pulse since the electrons will continue to gyrate in the static magnetic field until the latter diminishes. In this case we have $\Omega=(B_0/\gamma_f)\omega$. Figure 4 shows the harmonic (n) dependence of the radiated power $d_\theta P_n/P$ as observed at $\theta=\pi/2$, i.e., in the plane of the electron orbit. The other parameters are the same as that for Figs. 1(b) (dashed curve) and 1(c) (solid curve), respectively. We have as critical frequency $\Omega_c=324\Omega=27\omega$ and radiated power $P=822\hbar\omega^2$ for the dashed curve, and $\Omega_c=1.5\times10^5\Omega=952\omega$ and $P=1.7\times10^4\hbar\omega^2$ for the solid curve. The radiation time is now determined by the magnetic field lifetime, which at present is at least five orders of magnitude longer than that of the laser pulse.

V. DISCUSSION

In this paper, we have considered electron acceleration and high-harmonic generation by an intense short linearly polarized laser pulse in an external magnetic field parallel to that of the light wave. It is shown that an ionized electron can be strongly energized, and its maximum velocity is obtained. The sawtooth maximum speed versus magnetic field profile agrees well with that from the quantum theory [17]. In fact, the other characteristics of the interaction are also qualitatively similar. Thus the present results show that the acceleration process is mainly classical and the atomic potential does not significantly contribute to the evolution of the electron motion. The physical mechanism of electron acceleration here also differs from that of Apollonov and coworkers [20–22], in that here it does not rely on the creation of a quasistatic acceleration field and is thus effective for short pulses. We also found that for higher laser intensities, the sawtooth profile is replaced by a flat-top one and electron acceleration to relativistic speeds can be achieved with a weaker external magnetic field, which also results in the retention of much of the gained energy by the electron in the form of cyclotron motion even after the passing of the laser pulse. Thus a very large amount of radiation at short wavelengths can be produced.

As much of the electron energization is in its cyclotron motion, the overall space required for the acceleration is only of the order of the laser wavelength and cyclotron radius. Furthermore, since high-intensity magnetic fields can be of much longer duration than that of the short laser pulse under consideration here, the total time for high-harmonic generation is given by the lifetime of the magnetic field instead of the laser pulse. Compared to that in existing laser-driven and conventional synchrotron radiation schemes, the energized electrons here have less kinetic energy but their radiation lifetime is much longer than the laser pulse, and their typical orbit radius is seven orders of magnitude smaller than that in a synchrotron. The present process can therefore be useful as an efficient and compact source of synchrotron radiation.

ACKNOWLEDGMENTS

This work was supported by the National High-Tech ICF Committee of China, the National Natural Sciences Foundation of China (Contract No. 19974058), the Sonderforschungsbereich, and the Chinese Academy of Sciences – Max Planck Gesselschaft Exchange Program.

[1] E. Esarey, S.K. Ride, and P. Sprangle, Phys. Rev. E **48**, 3003 (1993).
[2] E.E. Fill, J. Opt. Soc. Am. B **11**, 2241 (1994).
[3] E. Esarey, P. Sprangle, M. Pilloff, and J. Krall, J. Opt. Soc. Am. B **12**, 1695 (1995).
[4] E. Esarey, P. Sprangle, and J. Krall, Phys. Rev. E **52**, 5443 (1995).
[5] Ch. Spielmann *et al.*, Science **278**, 661 (1997).
[6] E. Esarey *et al.*, IEEE J. Quantum Electron. **33**, 1879 (1997).
[7] P. Gibbon, IEEE J. Quantum Electron. **33**, 1915 (1997).
[8] Z. Chang *et al.*, Phys. Rev. Lett. **79**, 2967 (1997).
[9] G. Malka, E. Lefebvre, and J.L. Miquel, Phys. Rev. Lett. **78**, 3314 (1997).
[10] W. Yu, M.Y. Yu, J.X. Ma, and Z. Xu, Phys. Plasmas **5**, 406

(1998).
[11] F. Krausz, T. Brabec, M. Schurer, and C. Spielmann, Opt. Photonics News **9**, 46 (1998).
[12] M. Schnurer, *et al.*, Phys. Rev. Lett. **80**, 3236 (1998).
[13] P. Mora and B. Quesnel, Phys. Rev. Lett. **80**, 1351 (1998); B. Quesnel and P. Mora, Phys. Rev. E **58**, 3719 (1998).
[14] X. Li, Phys. Rev. E **59**, 6048 (1999).
[15] A.L. Troha *et al.*, Phys. Rev. E **60**, 926 (1999).
[16] W. Yu *et al.*, Phys. Rev. E **61**, R2220 (2000).
[17] R.E. Wagner, Q. Su, and R. Grobe, Phys. Rev. Lett. **84**, 3282 (2000).
[18] B. Hafizi *et al.*, Phys. Rev. E **62**, 4120 (2000).
[19] D. Umstadter, Phys. Plasmas **8**, 1774 (2001), and references therein.
[20] V.V. Apollonov *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **47**, 77 (1988) [JETP Lett. **44**, 91 (1988)]; Zh. Éksp. Teor. Fiz. **97**, 1498 (1990) [Sov. Phys. JETP **70**, 846 (1990)].
[21] V.V. Apollonov and M.V. Fedorov, Laser Phys. **1**, 662 (1991).
[22] V.V. Apollonov, M.V. Fedorov, A.M. Prokhorov, and A.G. Suzdaltsev, IEEE J. Quantum Electron. **28**, 265 (1992).
[23] P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading, MA, 1969), Sec. 4.5.2.
[24] E.S. Sarachik and G.T. Schappert, Phys. Rev. D **1**, 2738 (1970).